ON MAGNETIC ATTITUDE CONTROL OF GRAVITY GRADIENT SATELLITES

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CERTIFICATE

This is to certify that the thesis entitled "On Magnetic Attitude Control of Gravity Gradient Satellites" by Parmeshwar Raut is a record of work carried out under my supervision and has not been submitted elsewhere for a degree.

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ABSTRACT

Utilization of the earth's magnetic field for attitude control of gravity gradient satellites is studied. A controller consisting of a single body-fixed onboard electromagnet is proposed for achieving simultaneous control of the three-dimensional rotational motion of the spacecraft. The Liapunov stability theorem is applied to the linearized system equations to synthesize a general control law for the dipole strength. A response analysis establishes the validity of the approach in orbits with different inclinations. The system offers enhanced reliability as no moving parts are involved. The semi-passive character of the system promises an increased operational life-span for the satellite.

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LIST OF SYMBOLS

A	area enclosed by the coil
A_i	error function constants, $i = 1,5$
$\bar{B}$	geomagnetic induction vector
B_x, B_y, B_z, B_{xn}, B_{yn}, B_{zn}	components of $\bar{B}$ along the x,y,z and x_n, y_n, z_n axes, respectively
D_m	geomagnetic dipole moment, M_e/R^3; $M_e = 8.1 \times 10^{25}$ Gauss-cm^3
I	magnitude of current flowing in the coil
I_x, I_y, I_z	principal moments of inertia of the satellite
K	controller parameter
K_1	$(I_y - I_z)/I_x$
K_2	$(I_z - I_x)/I_y$
$\bar{M}_m$	magnetic torque vector
M_x, M_y, M_z	components of total torque about x,y,z axes, respectively
M_{gx}, M_{gy}, M_{gz}	components of gravity gradient torque about x,y,z axes, respectively
M_{mx}, M_{my}, M_{mz}	components of magnetic torque about x,y,z axes, respectively
O	centre of the earth
P	perigee
$\bar{P}$	applied magnetic dipole vector
P_g, P_m	geographic and geomagnetic polar axes, respectively
Q	transformation matrix

R	distance between the satellite centre of mass and the centre of the earth
S	centre of mass of the satellite
V	Liapunov function
X,Y,Z	inertially fixed co-ordinate system with X along the **perigee and Z-axis normal** to the orbit plane
i	inclination of the orbit plane from the equatorial plane
$\bar{i},\bar{j},\bar{k}$	unit vectors along x,y,and z axes, respectively
n_e	rate of rotation of the earth
$\bar{p}$	unit vector along the normal to the plane of the coil
p_x,p_y,p_z	components of $\bar{p}$ along x,y and z axes,respectively.
u	magnetic moment of the electromagnet
u_o	maximum strength of the dipole
$\underline{x}$, $\underline{y}$	state vectors
x,y,z	principal body co-ordinates
x_o,y_o,z_o	rotating co-ordinate system with x_c along the local vertical and z_o along the normal to the orbit plane
x_1,y_1,z_1	intermediate body co-ordinates resulting from
x_2,y_2,z_2	rotations and about x_o and y_1 axes, respectively
x_n,y_n,z_n	inertial co-ordinate system with z_n normal to the orbit plane and x_n along the line of nodes referred to the equatorial plane
Ω	orbital velocity of the satellite in the circular orbit

Ω_m	angles between the vernal equinox and the line of nodes referred to the equatorial plane
α	controller parameter
β, γ, λ	yaw, roll and pitch angle, respectively
δ_1, δ_2	spherical co-ordinates of the axis normal to the coil
ε	geomagnetic polar axis declination, $\varepsilon=11.4^\circ$
η	$\omega+\theta$
θ	orbital angle
μ	power of the earth as a centre of force
σ	error function
ϕ	advance of the line of intersection of plane containing P_g, and P_m with the equatorial plane from the line of nodes
ω	argument of the perigee
ω_1, ω_2	characteristic roll-yaw frequencies
$\omega_x, \omega_y, \omega_z$	component of the angular velocity of the satellite along x,y and z axes, respectively
$(\dot{\ })$	d/dt
$(\ )'$	$d/d\theta$
$(\ )_o$	initial condition.

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1. INTRODUCTION

The general motion of a space vehicle is quite complex. It may be thought of as the superposition of two distinct modes of motion: the translation of the vehicle centre of mass and the rotation of the spacecraft about the centre of mass. The curvilinear translation of the mass centre relative to the primary attracting body describes the orbit of the satellite. The rotational motion about the mass centre, on the other hand, determines the orientation of the vehicle in space. It is also referred to as the attitude motion of the satellite.

The success of a vast majority of space applications, such as communications, weather, military, earth-resources, etc., depends on the ability of the spacecraft to maintain a preferred orientation in space. Unfortunately, a satellite initially oriented along the desired attitude deviates from it due to several reasons. These are: the gravity gradient effect, magnetic field interactions, aerodynamic forces, solar radiation pressure, uncompensated motion of internal machinery and micrometeorite impacts. Magnetic field interactions constitute a significant perturbing effect for satellites below 1600 km altitude. Solar radiation pressure represents the principal disturbing effect for high altitude

missions, e.g., in the geostationary orbit. The effect of aerodynamic forces is relatively unimportant for satellite altitudes above approximately 800 km.

For successful orientation of the satellite, an attitude control system is therefore required. The various methods available for attitude control may be classified as active, passive and semipassive according to the power consumption involved. The use of control elements such as gas jets, reaction wheels and gyros come under active methods of attitude control. The active methods require a large energy consumption which limits the life-span of the satellite. A high speed of response and accuracy can be obtained by these techniques.

Passive methods, on the other hand, do not involve any energy consumption. Gravity gradient stabilization and spin stabilization fall in this category. In gravity gradient stabilization, the values for the moments of inertia about the principal axes of the satellite are so chosen that the vehicle will have gravitational restoring torques about all the three axes. The vehicle then tends to orient itself with its axis of minimum moment of inertia along the local vertical and the maximum moment of inertia axis normal to the orbit plane. Spin stabilization technique is based on the fact that a spinning body exhibits gyroscopic stiffness against external torques. The satellite is spun about its axis of symmetry

and it tends to maintain its angular momentum vector fixed in inertial space. The passive methods, in general, lead to an increased operational lifetime of the satellite as no energy consumption is required. Their pointing accuracy capabilities, however, are limited.

The semipassive methods represent a compromise between the active and passive methods. The concept involves utilization of suitable controllers which interact with the space environment to generate control torques about the satellite centre of mass. The utilization of the aerodynamic forces, solar radiation pressure and magnetic field interactions, come under this class. In aerodynamic control, suitably designed control surfaces are extended from the satellite to intercept the relative atmospheric flow to obtain the control torques. Mirror-like surfaces may be used at high altitudes to generate control moments from solar radiation pressure. In magnetic attitude control system, electromagnets are used onboard to interact with the earth's magnetic field. By varying the onboard dipole strength and orientation, one can govern the magnetic torques about the centre of mass of the satellite. The energy consumption in these methods is limited to activating the controller elements. The semipassive methods provide a high pointing accuracy and a moderate speed of response. They neither involve any mass-expulsion nor the use of elements requiring

a large power consumption, and hence, promise an increased life-span for the satellite.

Generation of control torques through the interaction of onboard electromagnetic dipoles and the earth's magnetic field appears to be particularly attractive as the system reliability is enhanced by the elimination of moving parts. This thesis deals with some aspects of the magnetic attitude control of spacecraft. Important previous investigations in the area are briefly discussed next.

Brief Survey of Previous Work:

Libration damping of gravity-gradient spacecraft using passive hysteresis damping was considered by Alper and O'Neill[1] and Fischell[2]. The authors suggested using rods of high permeability magnetic alloys for damping the attitude motions. The response rates of the hysteresis dampers, however, are extrmely slow. The concept of closed-loop magnetic torquing and its feasibility was suggested by White et al.[3] Banium and Mackison[4] applied the sample and hold concept to a gravity gradient satellite employing three mutually perpendicular onboard electromagnets for effecting three-axis attitude damping. Modi and Pande[5] investigated the feasibility of three axis nutation damping and attitude control of both spinning and nonspinning satellites using a hybrid magnetic-solar control system. The authors

considered two magnetic controller models involving a rotatable electromagnet and two fixed electromagnets for spin axis orientation, and the use of solar radiation pressure for simultaneous control of the satellite pitch attitude.

The problem of maintaining the spin-axis of a spinning satellite normal to the orbit plane has been a subject of considerable discussion. Ergin and Wheeler[6] proposed an autonomous magnetic attitude control system for the purpose employing a dipole along the pitch axis. Subsequently, Wheeler[7] presented a feedback control law including provisions for active magnetic nutation damping as well as directing the spin axis of an axisymmetric spinning satellite in a circular orbit toward any direction in space. The stability of the control law was demonstrated via the method of Liapunov. However, a torque-free environment was assumed. Renard[8] obtained an open loop control law that requires quarter orbit switching for near polar orbits. Flügge Lotz, and Maltz[9] considered a contactor control system to stabilize the attitude of a spinning space vehicle and obtained the system response using a linear switching criterion. The authors accounted for gravity torques during the analysis but the system stability was not demonstrated. Sonnabend[10] and Dahl and Foxman[11] developed precession control laws that utilize a roll axis dipole. Tossman[12] proposed a

control system for magnetic nutation and despin control which utilizes the concept of enhanced magnetic hysteresis damping. Vrablik et al.[13,14] proposed an attitude control system for the spin-axis orientation of the LES-4 and LES-5 spacecraft using a system of electromagnets as the sole source of corrective torque at the synchronous altitude. Sorenson[15] developed magnetic pointing system applicable to spacecraft with a limited attitude determination capability. Shigehara[16] developed a control law based on the asymptotic stability criterion for the spin axis and spin rate control using dipoles along the spin axis and perpendicular to it, respectively. Collins and Bonello[17] discussed several magnetic control schemes that provide nutation as well as precession control. Alfriend[18] proposed a closed loop control law for a dual spin satellite using a dipole parallel to the spin axis. Alfriend and Stickler[19] developed a three axis closed loop attitude control system for Earth observatory momentum bias spacecraft for initial acquisition, precession control, nutation damping and pitch axis momentum control.

It is apparent that the possibility of achieving simultaneous control of the roll, yaw and pitch motions by means of a single, body-fixed electromagnet has not been investigated so far. On the other hand, it would represent a significant reduction of the hardware design without any

loss of system reliability. This thesis proposes and develops a magnetic attitude control system for gravity gradient satellites involving a single electromagnet with a body-fixed orientation in the spacecraft principal axis system. A control law for the dipole strength, promising asymptotically stable operation, is synthesized and the performance of the control system is examined through a response analysis of the system.

2. FORMULATION OF THE PROBLEM

2.1 Geometry of Motion and Controller Configuration:

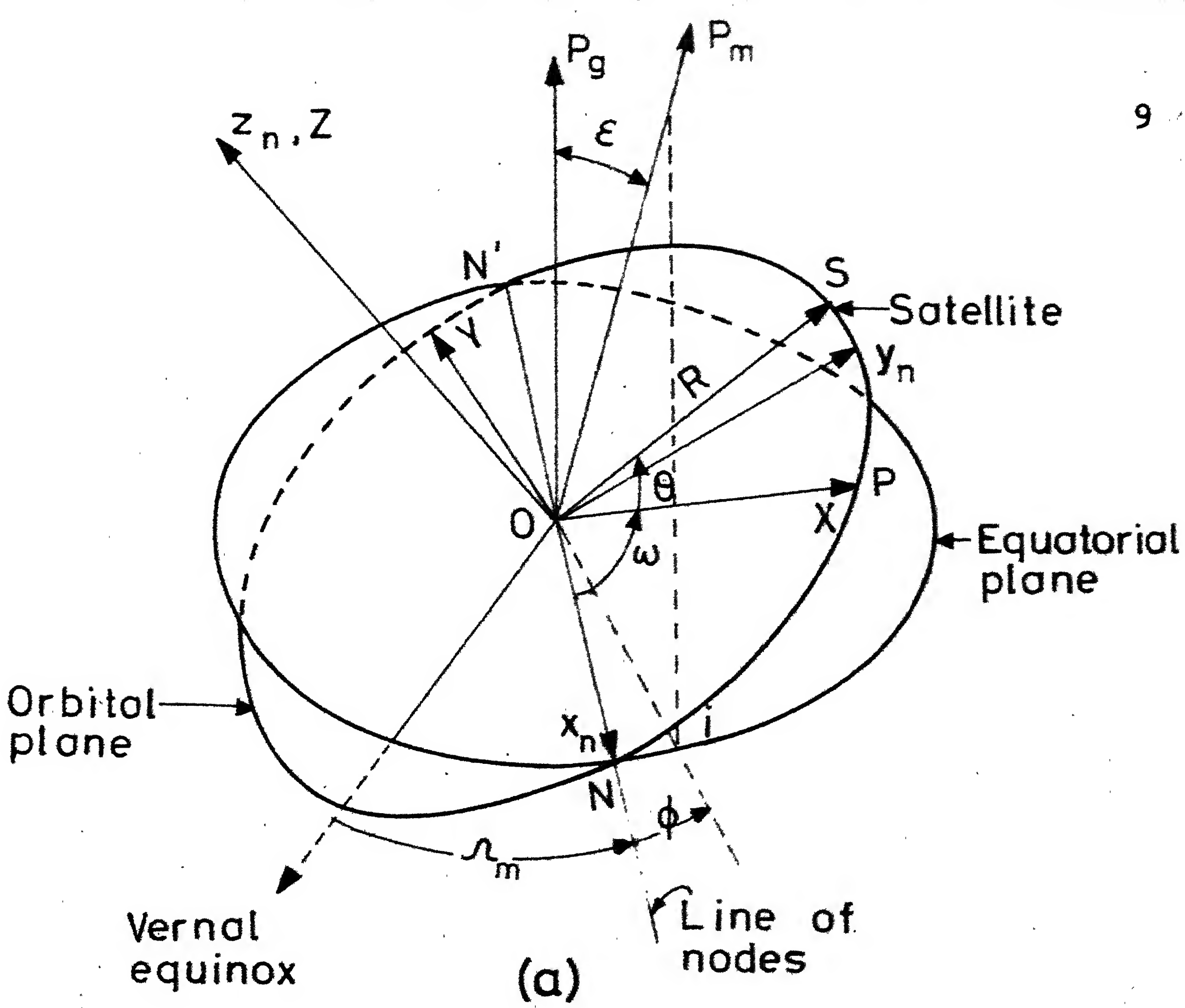
Fig.2.1a shows the orbital motion of an unsymmetrical satellite with its centre of mass S moving in a Keplerian orbit around the centre of the earth O. P_g and P_m indicate the geographic and magnetic axes of the earth directed from the earth's centre to its geographic and magnetic North poles, respectively. The two are offset by an angle ε ($\approx 11.4°$). As the earth spins about its geographic axis, the intersection of the line of intersection of the plane containing P_g and P_m with the equatorial plane advances uniformly with time.

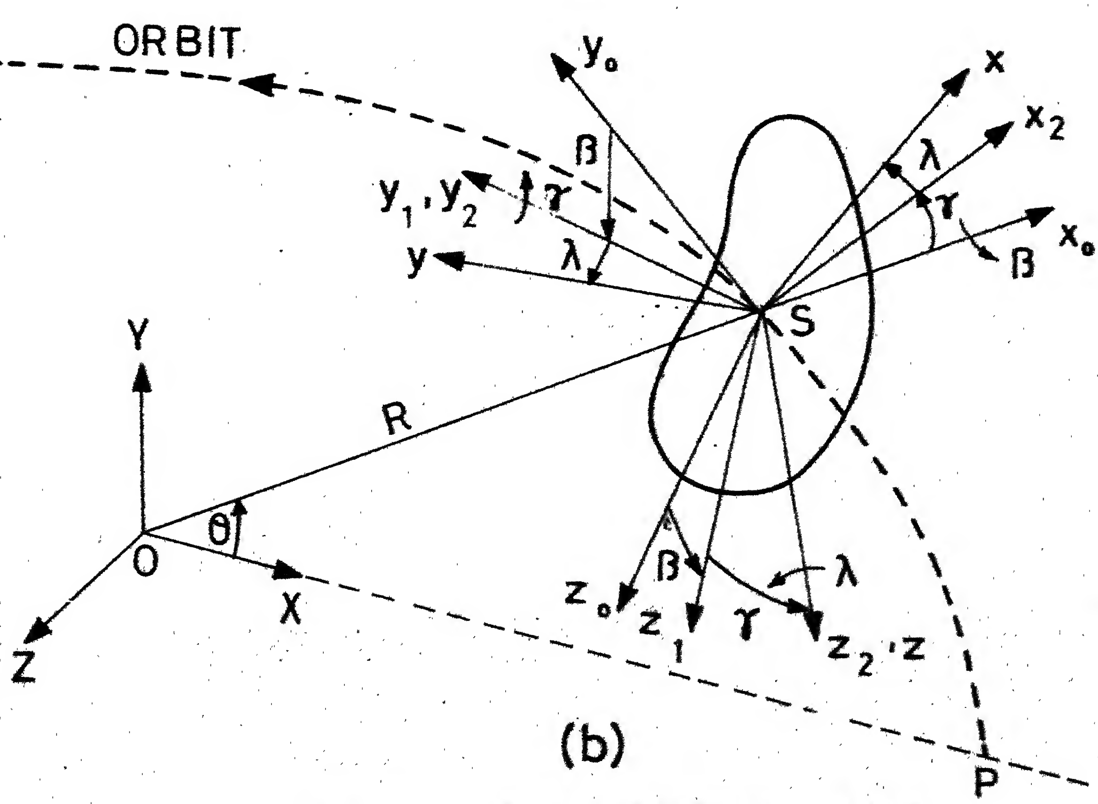
The rotational motion of the satellite about its centre of mass is shown in Fig.2.1b. To describe the satellite attitude relative to the rotating orbital co-ordinate system x_o, y_o, z_o, the following sequence of finite rotations may be employed:

(i) the rotation β about the x_o axis bringing x_o, y_o, z_o to x_1, y_1, z_1,

(ii) the rotation γ about the y_1 axis bringing x_1, y_1, z_1 to x_2, y_2, z_2, and finally,

(iii) the rotation λ about the z_2 axis bringing x_2, y_2, z_2 in coincidence with the body-fixed satellite principal axes x, y, z.





<Fig. 2.1 (a) Geometry of orbital motion of the satellite.

The rotations , and are referred to as the yaw, roll and pitch attitude angles, respectively. Any spatial orientation of the spacecraft may be uniquely described by allowing appropriate yaw, roll and pitch rotations in their proper sequence.

Fig.2.2 shows a schematic representation of the proposed magnetic controller which consists of a single electromagnet with its dipole moment $\bar{P}$. The magnitude of $\bar{P}$ depend upon the area enclosed by the current carrying coil and the curre passing through it. Its direction is determined by the direction of the current in the coil according to the right-handed screw rule. The unit vector $\bar{p}$ indicating the normal to the plane of coil is located by its spherical co-ordinates δ_1 and δ_2 relative to the principal axes x,y,z.

2.2 Equations of Motion:

The equations of motion for the spacecraft may be derived using either the Lagrangian or the Eulerian formulation. The overall system possesses five degrees of freedom, namely, R, θ, β, γ and λ. The generalized co-ordinates R, θ describe the orbital motion while β, γ and λ describe the attitude motion of the satellite. The two, in general, are coupled and this leads to a very complicated five degree-of-freedom system. However, it is well known that the influence of the attitude motion on

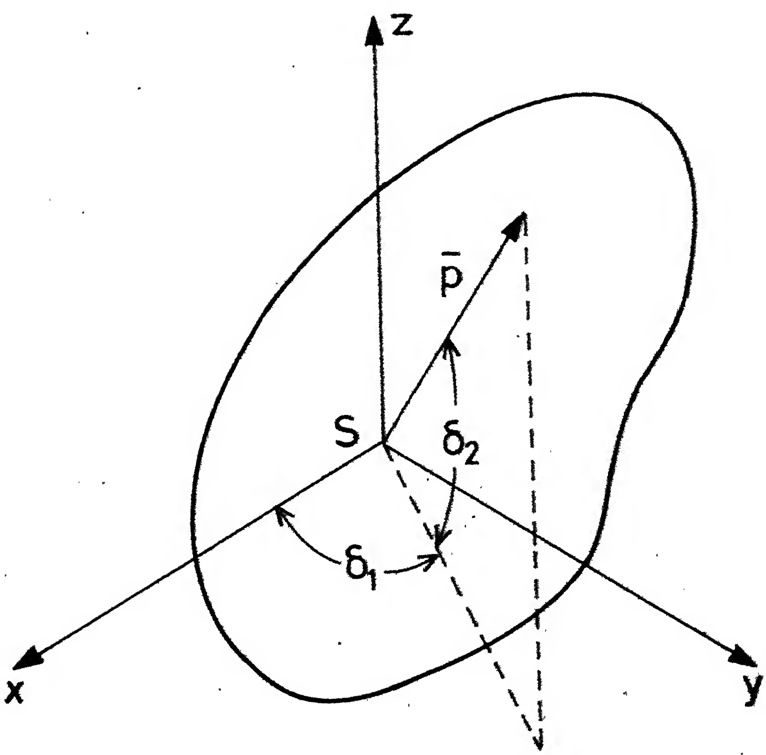


Fig. 2.2 Controller Configuration.

the orbital motion is negligible. Therefore, one may directly derive the equations of attitude motion and simplify the derivation significantly making use of the fact that the orbit remains essentially Keplerian. Presently the derivation for the case of a circular satellite orbit with angular speed $\dot{\theta} = \Omega$ is given using the Eulerian approach.

2.2.1 Eulerian formulation:

Euler's equations of motion for the rotational motion of a rigid body about its centre of mass are well known to be

$$I_x \dot{\omega}_x + (I_z - I_y) \omega_y \omega_z = M_x \qquad (2.1a)$$

$$I_y \dot{\omega}_y + (I_x - I_z) \omega_z \omega_x = M_y \qquad (2.1b)$$

$$I_z \dot{\omega}_z + (I_y - I_x) \omega_x \omega_y = M_z \qquad (2.1c)$$

where the moments of inertia, angular velocity components and the external torque components are referred to the principal axes x.y,z.

The external torque components M_x, M_y, M_z consist of the contributions from the gravity gradient torque and the magnetic torque, i.e.,

$$M_x = M_{gx} + M_{mx} \qquad (2.2a)$$

$$M_y = M_{gy} + M_{my} \qquad (2.2b)$$

$$M_z = M_{gz} + M_{mz} \tag{2.2c}$$

The gravitational torque about the centre of mass arises due to the variation of the gravity field across the dimensions of the satellite. Its components are given by[20]

$$M_{gx} = 3(\frac{\mu}{R^3})(I_y - I_z)\sin\gamma\ \cos\gamma \sin\lambda \tag{2.3a}$$

$$M_{gy} = 3(\frac{\mu}{R^3})(I_x - I_z)\sin\gamma\ \cos\gamma\cos\lambda \tag{2.3b}$$

$$M_{gz} = 3(\frac{\mu}{R^3})(I_x - I_y)\sin\lambda\ \cos\lambda\ \cos^2\gamma \tag{2.3c}$$

where terms of the order (satellite dimension/R)^5 and higher are ignored compared to unity in its expansion.

The magnetic torque generated by the interaction of the onboard electromagnet with the earth's magnetic field may be written as

$$\bar{M}_m = \bar{P} \times \bar{B} = u\ \bar{p} \times \bar{B} \tag{2.4}$$

where

$$u = I\ A \tag{2.5}$$

with I representing the current and A the total area enclosed by the current carrying coil. The unit vector $\bar{p}$ along the normal to the plane of the coil is simply

$$\bar{p} = p_x\ \bar{i} + p_y\ \bar{j} + p_z\ \bar{k} \tag{2.6}$$

where

$$p_x = \cos\delta_1 \cos\delta_2 \tag{2.7a}$$

$$p_y = \sin\delta_1 \cos\delta_2 \tag{2.7b}$$

$$p_z = \sin\delta_2 \tag{2.7c}$$

The geomagnetic induction vector $\overline{B}$ may be expressed in terms of its components in the x_n, y_n, z_n system by assuming the earth to be a canted magnetic dipole as[21]

$$B_{xn}/D_m = (\tfrac{3}{2})\sin i \sin 2\eta \cos\varepsilon + (\tfrac{1}{2})[\cos\phi + (\tfrac{3}{2})\{(1+\cos i)\cos(2\eta-\phi) + (1-\cos i)\cos(2\eta+\phi)\}]\sin\varepsilon \tag{2.8a}$$

$$B_{yn}/D_m = (\tfrac{1}{2})\sin i(1-3\cos 2\eta)\cos\varepsilon + (\tfrac{1}{2})[\cos i \sin\phi + (\tfrac{3}{2})\{(1+\cos i)\sin(2\eta-\phi)+(1-\cos i)\sin(2\eta+\phi)\}]\sin\varepsilon \tag{2.8b}$$

$$B_{zn}/D_m = -\cos i \cos\varepsilon + \sin i \sin\varepsilon \sin\phi \tag{2.8c}$$

where the geomagnetic dipole moment D_m is

$$D_m = M_e/R^3 \tag{2.9}$$

with

$$M_e = 8.1 \times 10^{25} \text{ Gauss-cm}^3.$$

Note that the angle ϕ varies due to the earth's rotation according to $\dot{\phi} = n_e$. With θ as the independent variable in the circular orbit, this gives

$$\phi = (n_e / \Omega) + \phi_o \tag{2.10}$$

The present analysis ignores any nodal regression as a dynamic effect which is equivalent to assuming the earth to be gravitationally spherical. If desired, it may be accounted for by appropriately updating ϕ_o after every few orbits. The body components of $\bar{B}$ may be obtained from the components along the x_n, y_n, z_n system through a series of co-ordinate transformations. The final expression is obtained as

$$\begin{bmatrix} B_x \\ \\ B_y \\ \\ B_z \end{bmatrix} = \begin{bmatrix} \cos\lambda\cos\gamma\cos\eta - \sin\lambda\cos\beta\sin\eta - \sin\gamma\cos\lambda\sin\beta\sin\eta & \cos\lambda\cos\gamma\sin\eta + \sin\lambda\cos\beta\cos\eta + \sin\gamma\cos\lambda\sin\beta\cos\eta & \sin\lambda\sin\beta - \sin\gamma\cos\lambda\cdot\cos\beta \\ -\sin\lambda\cos\gamma\cos\eta - \cos\lambda\cos\beta\sin\eta + \sin\lambda\sin\beta\sin\gamma\cdot\sin\eta & -\sin\lambda\cos\gamma\sin\eta + \cos\lambda\cos\beta\cos\eta - \sin\lambda\sin\gamma\sin\beta\cdot\cos\eta & \cos\lambda\sin\beta + \sin\lambda\sin\gamma\cdot\cos\beta \\ \sin\gamma\cos\eta + \cos\gamma\sin\beta\sin\eta & \sin\gamma\sin\eta - \cos\gamma\sin\beta\cos\eta & \cos\beta\cos\gamma \end{bmatrix} \begin{bmatrix} B_{xn} \\ \\ B_{yn} \\ \\ B_{zn} \end{bmatrix} \tag{2.11}$$

The magnetic torque $\bar{M}_m$ is thus obtained in terms of its components in the principal axis system by substituting

Eqns.(2.11) and (2.7) in Eqn.(2.4) and carrying out the vector cross product. Subsequent substitution of these components into Eqns. (2.2) finally leads to the desired total external torque expressions.

The determination of the kinematical relationships for the body components of the angular velocity is fairly straightforward. Again, the use of a number of co-ordinate transformations is involved, finally leading to

$$\omega_x = (\dot{\gamma} + \Omega \sin\beta)\sin\lambda + (\dot{\beta}\cos\gamma - \Omega\cos\beta \sin\gamma)\cos\lambda \qquad (2.12a)$$

$$\omega_y = (\dot{\gamma} + \Omega \sin\beta)\cos\lambda - (\dot{\beta}\cos\gamma - \Omega\cos\beta\sin\gamma)\sin\lambda \qquad (2.12b)$$

$$\omega_z = \dot{\lambda} + \dot{\beta} \sin\gamma + \Omega \cos\beta \cos\gamma \qquad (2.12c)$$

Substituting Eqns. (2.1.2) in Eqns. (2.1) and introducing the total torque components, the equations of motion governing the attitude motion result. For convenience, the orbital angle $\theta = \Omega t$ may be introduced as the independent variable. The nondimensionalized equations of motion are finally obtainned as

$$
\begin{aligned}
&\beta'' \cos\gamma - [K_1\{ 3\sin\gamma\cos\gamma \sin\lambda + (\lambda'\gamma' + \lambda'\sin\beta)\cos\lambda \\
&\quad + \lambda'\sin\lambda(\cos\beta\sin\gamma - \beta'\cos\gamma) + (\beta'\gamma' + \beta'\sin\beta)\sin\gamma\cos\lambda \\
&\quad + \beta'\sin\lambda(\cos\beta\sin^2\gamma - \beta'\sin\gamma\cos\gamma) \\
&\quad + (\gamma' + \sin\beta)\cos\beta\cos\gamma\cos\lambda + (\cos^2\beta\cos\gamma\sin\gamma \\
&\quad - \beta'\cos\beta\cos^2\gamma)\sin\lambda\}\cos\lambda + K_2\{ 3\sin\gamma\cos\gamma\cos\lambda \\
&\quad - (\lambda'\gamma' + \lambda'\sin\beta)\sin\lambda + \lambda'\cos\lambda(\cos\beta\sin\gamma - \beta'\cos\gamma) \\
&\quad - (\beta'\gamma' + \beta'\sin\beta)\sin\gamma\sin\lambda + \beta'\cos\lambda(\cos\beta\sin^2\gamma \\
&\quad - \beta'\sin\gamma\cos\gamma) - (\gamma' + \sin\beta)\cos\beta\cos\gamma\sin\lambda \\
&\quad + (\cos^2\beta\cos\gamma\sin\gamma - \beta'\cos\beta\cos^2\gamma)\cos\lambda\}\sin\lambda \\
&\quad - \beta'\sin\beta\sin\gamma + \beta'\gamma'\sin\gamma + \gamma'\cos\beta\cos\gamma - \lambda'\sin\beta - \lambda'\gamma'] \\
&= \frac{1}{\Omega^2 I_x}(p_y B_z - p_z B_y)u\cos\lambda \\
&\quad + \frac{1}{\Omega^2 I_y}(p_z B_x - p_x B_z)u\sin\lambda
\end{aligned}
\tag{2.13a}
$$

$$\gamma'' - K_1 \{ 3\sin\gamma \cos\gamma \sin\lambda + (\lambda'\gamma' + \lambda' \sin\beta)\cos\lambda +$$

$$+ \lambda'\sin\lambda (\cos\beta\sin\gamma - \beta' \cos\gamma) + (\beta'\gamma' + \beta' \sin\beta)\sin\gamma \cos\lambda$$

$$+ \beta' \sin\lambda(\cos\beta \sin^2\gamma - \beta' \sin\gamma \cos\gamma) + (\gamma' + \sin\beta)\cos\beta \cos\gamma\cos\lambda$$

$$+ (\cos^2\beta\cos\gamma \sin\gamma - \beta' \cos\beta \cos^2\gamma)\sin\lambda\} \sin\lambda$$

$$+ K_2\{ 3\sin\gamma \cos\gamma \cos\lambda - (\lambda'\gamma' + \lambda'\sin\beta)\sin\lambda$$

$$+ \lambda'\cos\lambda(\cos\beta \sin\gamma - \beta' \cos\gamma) - (\beta'\gamma' + \beta'\sin\beta)\sin\gamma \sin\lambda$$

$$+ \beta'\cos\lambda(\cos\beta \sin^2\gamma - \beta' \sin\gamma \cos\gamma) - (\gamma' + \sin\beta)\cos\beta \cos\gamma \sin\lambda$$

$$+ (\cos^2\beta \cos\gamma \sin\gamma - \beta'\cos\beta \cos^2\gamma)\cos\lambda\} \cos\lambda$$

$$+ \beta' \cos\beta + \lambda'\cos\beta \sin\gamma - \beta'\gamma'\cos\gamma$$

$$= \frac{1}{\Omega^2 I_x} \{(p_y B_z - p_z B_y) u \sin\lambda\}$$

$$+ \frac{1}{\Omega^2 I_y} \{(p_z B_x - p_x B_z) u \cos\lambda\} \qquad (2.13b)$$

$$\lambda'' + \left(\frac{K_1 + K_2}{1 + K_1 K_2}\right) [3\sin\lambda\cos\lambda\cos^2\gamma + (\gamma' + \sin\beta)^2 \sin\lambda\cos\lambda$$

$$- (\gamma' + \sin\beta)(\cos\beta\sin\gamma - \beta'\cos\gamma)\cos 2\lambda$$

$$- (\cos\beta \sin\gamma - \beta' \cos\gamma)^2 \sin\lambda \cos\lambda] - \beta' \sin\beta \cos\gamma$$

$$- \gamma'\cos\beta \sin\gamma + \beta'\gamma' \cos\gamma + \beta'' \sin\gamma$$

$$= \frac{1}{\Omega^2 I_z} [p_x B_y - p_y B_x] u \qquad (2.13c)$$

where the parameters K_1 and K_2 are defined as

$$K_1 = \frac{I_y - I_z}{I_x} \qquad (2.1.4a)$$

$$K_2 = \frac{I_z - I_x}{I_y} \qquad (2.14b)$$

3. CONTROL SYNTHESIS

The objective of this chapter is to arrive at a suitable control strategy for the magnetic moment of the onboard electromagnet. The general problem of establishing a control policy for the control variable u is rather formidable in view of the complexity of the governing system of equations. An approach based on small amplitude motion near the equilibrium configuration of the system is therefore adopted.

3.1 System Equilibrium and Stability:

In absence of the control u, it is observed that $\beta = \gamma = \lambda = 0$ represents an identical solution of Eqns.(2.13). Physically, it implies that in absence of any external disturbances the principal axes x,y,z remain aligned with the orbital co-ordinates x_o, y_o, z_o, respectively. This corresponds to an earth-pointing spacecraft attitude. Due to the effect of disturbances such as might result from initial misalignment or due to micrometeorite impacts, the attitude would deviate from its desired configuration. Linearizing the equations of motion about the equilibrium point, they simplify to:

$$\beta'' - K_1 \beta - (1+K_1)\gamma' = \frac{1}{\Omega^2 I_x}(p_y B_z - p_z B_y)u \qquad (3.1a)$$

$$\gamma'' + 4K_2\gamma - (K_2-1)\beta' = \frac{1}{\Omega^2 I_y}(p_z B_x - p_x B_z)u \tag{3.1b}$$

$$\lambda'' + 3(K_1+K_2)/(1+K_1K_2) = \frac{1}{\Omega^2 I_z}(p_x B_y - p_y B_x)u \tag{3.1c}$$

Note that the components of the earth's magnetic induction vector B_x, B_y, B_z are retained as such because these can be directly sensed by means of three mutually perpendicular magnetometers aboard the vehicle.

The conditions for the stability of the equilibrium of an uncontrolled satellite can be obtained easily from Eqns. (3.1) by letting $u = 0$ and examining the characteristic equation

$$\{ s^4 + (1+3K_2-K_1K_2)s^2 - 4K_1K_2 \}\{s^2 + \frac{3(K_1+K_2)}{(1+K_1K_2)}\} = 0 \tag{3.2}$$

The equation appears in the factorized form as the pitch motion of an uncontrolled satellite decouples from the roll-yaw motion near the equilibrium position. It is apparent that one can at best expect stability in the sense of Liapunov corresponding to purely imaginary roots of the characteristic equation. Also, because of the constraints on the inertia properties of a rigid body given by

$$I_x + I_y > I_z, \quad I_y + I_z > I_x, \quad I_z + I_x > I_y \tag{3.3}$$

the parameters K_1 and K_2 are limited to the range

$$-1 < K_1 < 1, \quad -1 < K_2 < 1, \quad K_1 K_2 > 0 \tag{3.4}$$

In view of these inequalities, the range of K_1, K_2 over which stable equilibrium exists can be expressed in the K_1-K_2 space[22] as shown in Fig. 3.1. Normally, the moments of inertia of a gravity gradient satellite are designed such that the system configuration in the K_1, K_2 space is located within the stable range shown. In the following, the satellite mass distribution is assumed to correspond to a nominally stable configuration.

3.2 Control Law Derivation:

The objective of the magnetic controller is to provide attitude control torques which maintain the earth pointing attitude of the satellite. A suitable control law, developing appropriate torques in all the three degrees of freedom, is presently synthesized via the second method of Liapunov based on the linearized equations of motion. The construction of a suitable Liapunov function becomes relatively easy by expressing the coupled roll-yaw equations in the state-variable form.

Letting $x_1 = \beta$, $x_2 = \beta'$, $x_3 = \gamma$, $x_4 = \gamma'$, Equations (3.1a and b) can be expressed in the state-variable form as

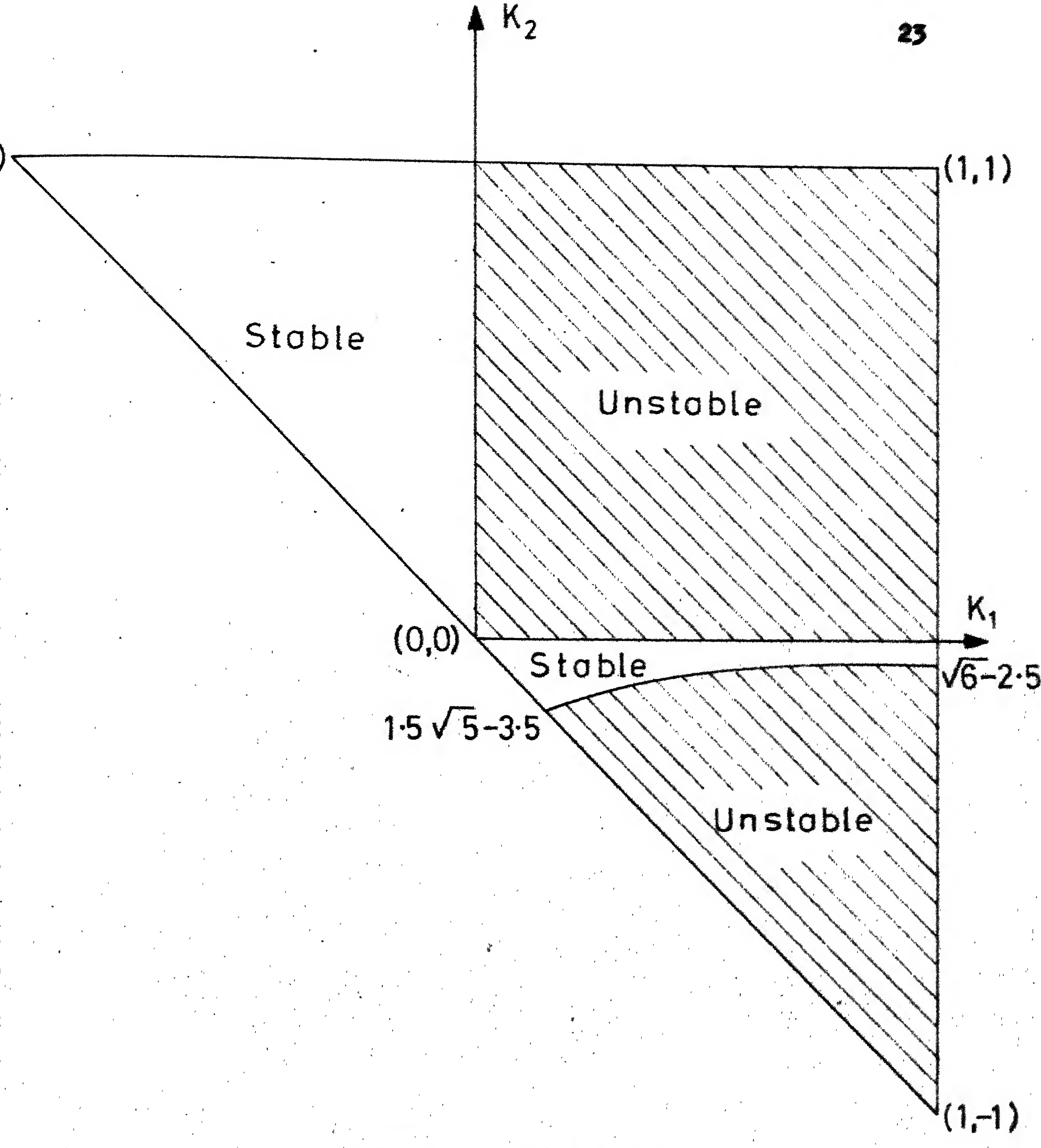


Fig.3.1 Stability diagram in the K_1-K_2 space.

$$\begin{bmatrix} x_1' \\ x_2' \\ x_3' \\ x_4' \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ K_1 & 0 & 0 & (1+K_1) \\ 0 & 0 & 0 & 1 \\ 0 & -(1-K_2) & -4K_2 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 \\ \dfrac{p_yB_z-p_zB_y}{\Omega^2 \; I_x} \\ 0 \\ \dfrac{p_zB_x-p_xB_z}{\Omega^2 \; I_y} \end{bmatrix} u \qquad (3.5)$$

A transformation matrix which uncouples the roll-yaw motions of an uncontrolled satellite can be found using methods of linear system theory. One such transformation is described by

$$\underline{y} = \underline{Q}\,\underline{x} \qquad (3.6)$$

where

$$Q = \begin{bmatrix} -K_1/\omega_1 & 0 & 0 & \dfrac{\omega_1(\omega_2^2-4K_2)}{4K_2(1-K_2)} \\ 0 & 1 & \dfrac{4K_2-\omega_2^2}{1-K_2} & 0 \\ 0 & \dfrac{\omega_2(\omega_1^2+K_1)}{K_1(1+K_1)} & \dfrac{4K_2}{\omega_2} & 0 \\ \dfrac{\omega_1^2+K_1}{1+K_1} & 0 & 0 & 1 \end{bmatrix} \qquad (3.7)$$

Here, the characteristic frequencies ω_1, ω_2 of the roll-yaw motion may be obtained as the solution of the equation

$$\omega^4 - (1 + 3K_2 - K_1K_2)\,\omega^2 - 4K_1K_2 = 0 \qquad (3.8)$$

When the transformation described by Eqns.(3.6) is applied to Eqn.(3.5), only the coupling in the terms involving the control variable would remain. Explicitly the result is found to be

$$\begin{bmatrix} y_1' \\ y_2' \\ y_3' \\ y_4' \end{bmatrix} = \begin{bmatrix} 0 & \omega_1 & 0 & 0 \\ -\omega_1 & 0 & 0 & 0 \\ 0 & 0 & 0 & \omega_2 \\ 0 & 0 & -\omega_2 & 0 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{bmatrix} + \begin{bmatrix} C_1\left(\dfrac{p_zB_x - p_xB_z}{\Omega^2\ I_y}\right) \\ \left(\dfrac{p_yB_z - p_zB_y}{\Omega^2\ I_x}\right) \\ C_2\left(\dfrac{p_yB_z - p_zB_y}{\Omega^2\ I_x}\right) \\ \left(\dfrac{p_zB_x - p_xB_z}{\Omega^2\ I_y}\right) \end{bmatrix} u \qquad (3.9)$$

where

$$C_1 = \frac{\omega_1(\omega_2^2 - 4K_2)}{4K_2(1-K_2)} \qquad (3.10a)$$

$$C_2 = \frac{(\omega_1^2 + K_1)\,\omega_2}{\ldots} \qquad (3.10b)$$

A Liapunov function for the complete system is now easily selected as

$$V = y_1^2 + y_2^2 + \alpha(y_3^2 + y_4^2) + \lambda^2 + \left(\frac{1+K_1K_2}{3(K_1+K_2)}\right) \lambda'^2 \tag{3.11}$$

where $\alpha > 0$ is an arbitrary constant.

Differentiating Eqn.(3.11) gives

$$V' = 2[\ y_1y_1' + y_2y_2' + \alpha(y_3y_3' + y_4y_4') + \lambda' + \left(\frac{1 + K_1K_2}{3(K_1+K_2}\right)\lambda''\lambda'\ ] \tag{3.12}$$

Substituting for the derivatives from Eqns. (3.9) and (3.1c), all terms not involving the control variable cancel out leading to

$$V' = \frac{2(1+\alpha C_2^2)}{\Omega^2\ I_x} u[(A_1\beta + A_4\gamma')(p_zB_x-p_xB_z) + (A_2\beta'+A_3\gamma) (p_yB_z - p_zB_y) + A_5(p_xB_y - p_yB_x)\lambda'] \tag{3.13}$$

where the constants A_1, A_2, A_3, A_4, A_5 are defined by

$$A_1 = \left(\frac{I_x}{I_y}\right)\left(\frac{1}{1+\alpha C_2^2}\right)\left(\frac{\alpha(\omega_1^2+K_1)}{1+K_1} - \frac{K_1\ C_1}{\omega_1}\right) \tag{3.14a}$$

$$A_2 = 1.0 \tag{3.14b}$$

$$A_3 = \left(\frac{1}{1+\alpha C_2^2}\right)\left(\frac{4K_2\alpha C_2}{\omega_2} + \frac{4K_2 - \omega_2^2}{(1-K_2)}\right) \tag{3.14c}$$

$$A_4 = \left(\frac{I_x}{I_y}\right)\left(\frac{1}{1+\alpha C_2^2}\right)(\alpha + C_1^2) \tag{3.14d}$$

$$A_5 = \left(\frac{I_x}{I_z}\right)\left(\frac{1}{1+\alpha\ C_2^2}\right)\left(\frac{1 + K_1K_2}{3(K_1+K_2)}\right) \qquad (3.14e)$$

Eqn.(3.13) may be utilized to determine a control law for u which promises asymptotic stability of the system by ensuring that $V' < 0$.

Now an error function σ is defined as

$$\sigma = (A\ \beta+A_4\gamma')(p_zB_x - p_xB_z) + (A_2\beta' + A_3\gamma)(p_yB_z - p_zB_y)$$

$$+ A_5(p_xB_y - p_yB_x)\ \lambda' \qquad (3.15)$$

and asymptotic stability is assured if

$$u = f(\sigma) \qquad (3.16)$$

where $f(\sigma)$ is any function with the property

$$\sigma f(\sigma) < 0 \qquad (3.17)$$

Eqns. (3.16) and (3.17) determine the control law for the onboard dipole moment. Both linear as well as nonlinear control laws follow from these. The linear form is written as

$$u = -K\ \sigma, \qquad K > 0 \qquad (3.18)$$

where K is a constant.

A typical nonlinear form would be the bang-bang control law

$$u = -u_o \text{ sgn } \sigma \qquad (3.19)$$

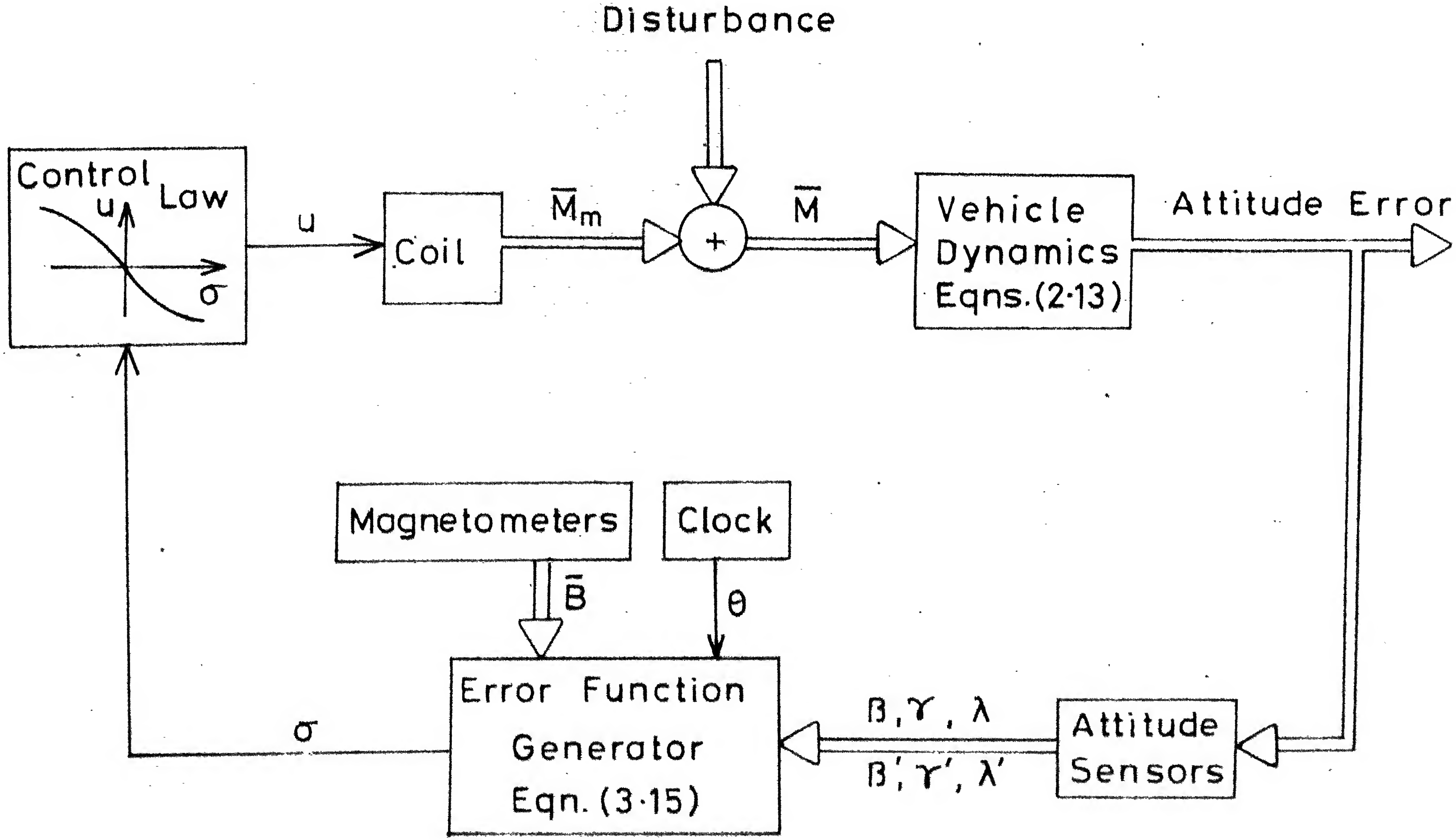


Fig. 3.2 Control flow diagram.

where u_o represents the maximum amplitude of the magnetic moment at which the controller operates all the time.

The mechanization of the proposed control system would involve the use of three magneto-meters oriented along three mutually perpendicular axes and attitude error as well as rate sensors. Besides, appropriate function generators would be required to continuously determine the control signals. A control flow diagram depicting the control system is shown in Fig.3.2.

4. RESULTS AND DISCUSSION

In order to evaluate the performance of the control system, the full nonlinear equations of motion (2.13) were solved in conjunction with the control law synthesized using the linearized system. As pointed out earlier, an analytical solution of the equations is not possible and therefore numerical integration was undertaken. A digital computer program using the Hamming's predictor-corrector method with a Runge-Kutta starter was developed. When the linear control law was considered, a step size of 0.5° gave results of sufficient accuracy over a wide range of system parameters.

As the system behaviour is characterized by a large number of parameters, it would take an execessive amount of computational effort to investigate the effect of each of them. Therefore, a specific satellite geometry was selected for the numerical study. The parameter values taken are:

$$\left.\begin{aligned} I_x &= 80 \text{ kg-m}^2 \\ I_y &= 120 \text{ kg-m}^2 \\ I_z &= 150 \text{ kg-m}^2 \end{aligned}\right\} \longrightarrow K_1 = -0.3750, \quad K_2 = 0.5833$$

Orbit inclination i = 45°

Orbit altitude R = 1000 km.

The associated control law was chosen to be the linear form (eqn. 3.18) and the controller parameters taken as:

$$\delta_1 = 45^\circ, \quad \delta_2 = 45^\circ, \quad K = 50, \quad \alpha = 1.5$$

These lead to the constants A_i $(i = 1,3,4,5)$ occurring in the error function to be

$$A_1 = 0.0079, \qquad A_3 = -0.0740$$

$$A_4 = 0.9007, \qquad A_5 = 0.5878$$

The response of the system subject to initial position and velocity errors was investigated first. These correspond to attitude errors which might result from an initial pointing misalignment and micrometeoritic impacts, respectively. The system response to a simultaneous initial error of 5° in roll, yaw and pitch is indicated in Fig. 4.1. The time-histories of the control variable u and the decay of the Liapunov function V are also shown. It is observed that all the three degrees of freedom are practically damped out ($< 0.1^\circ$) in about 5 orbits. The maximum onboard magnetic moment commanded during the process is found to be 1.38 amp-m^2. Similar results when the initial errors are doubled to 10° are presented in Fig.4.2. Here, all the three degrees of freedom settle within 0.1° in about 7 orbits. A comparison of the two cases suggests that the maximum magnetic moment required is almost double of the previous case.

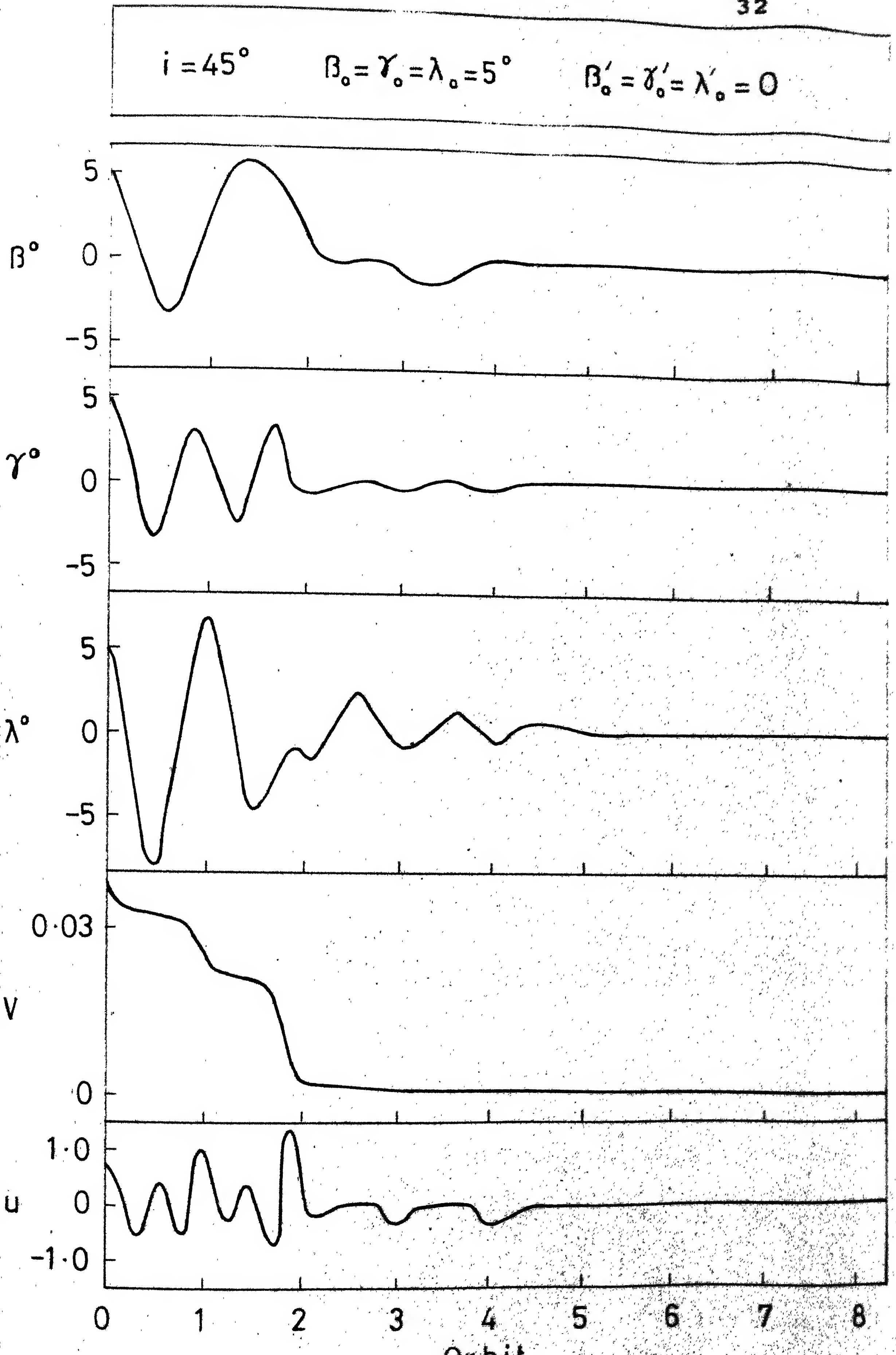


Fig. 4.1 System response and control history in an inclined orbit

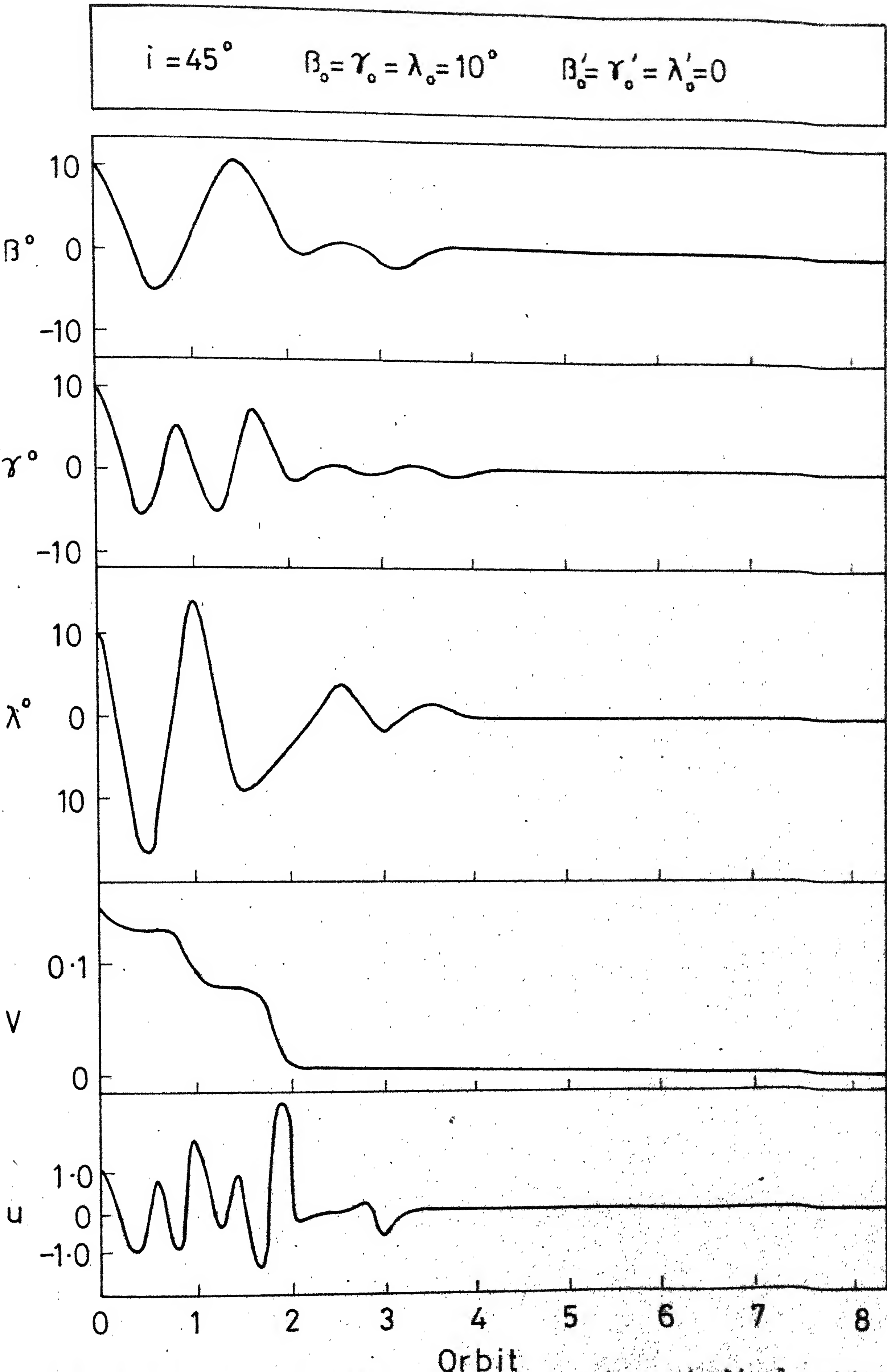


Fig 4.2 System response and control history in an inclined orbit

On the other hand, the system damping rate is higher when the initial errors are larger. This is also reflected in the nature of the decay rate of the V function which is a measure of the overall system damping. In this sense the behaviour of the present nonautonomous system appears to be similar to that of a linear time invariant system.

Figures 4.3 and 4.4 show the system response for the case of a correctly positioned satellite subject to impulsive disturbances. Again, simultaneous disturbances are considered in all the three degrees of freedom. The responses clearly indicate the capability of a single magnetic torques in countering the disturbance. It not only removes the resulting errors but the maximum attitude excursions during the process are also not too large. The associated dipole strength required is also quite small. The system settling times to within 0.1° are found to be approximately 5 and 12 orbits for the two initial conditions considered. It may be pointed out here that the disturbance values of 0.1 and 0.2 represent an extremely severe situation deliberately chosen to evaluate the controller performance under the most adverse conditions. In reality, the impulses due to micrometeorite impacts are at least an order of magnitude lower than these. Consequently, the performance of the system would be expected to be far superior to the responses presented here.

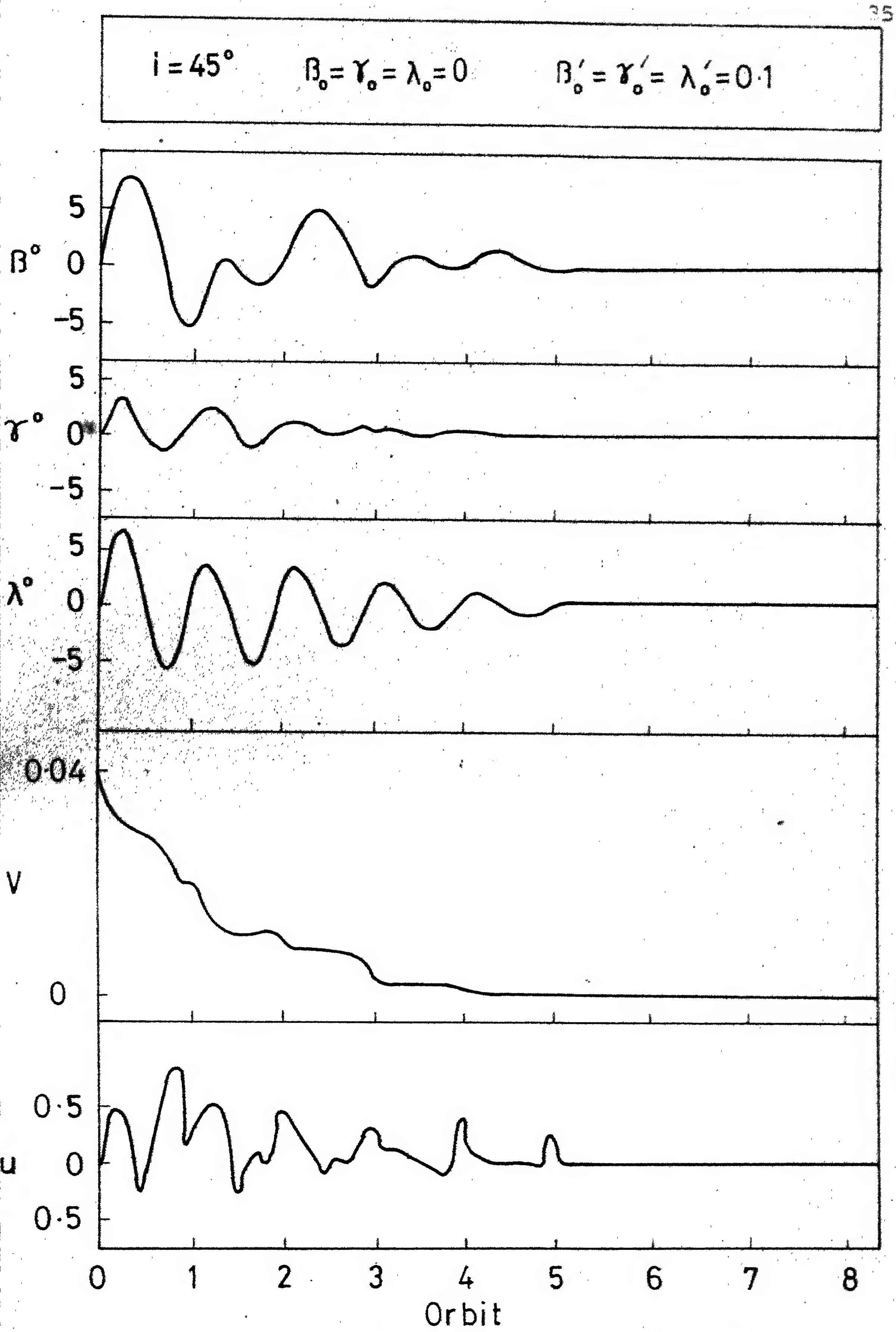


Fig.4.3 System response and control history in an inclined orbit ($i = 45^{o}$) with initial impulsive disturbance of 0.1.

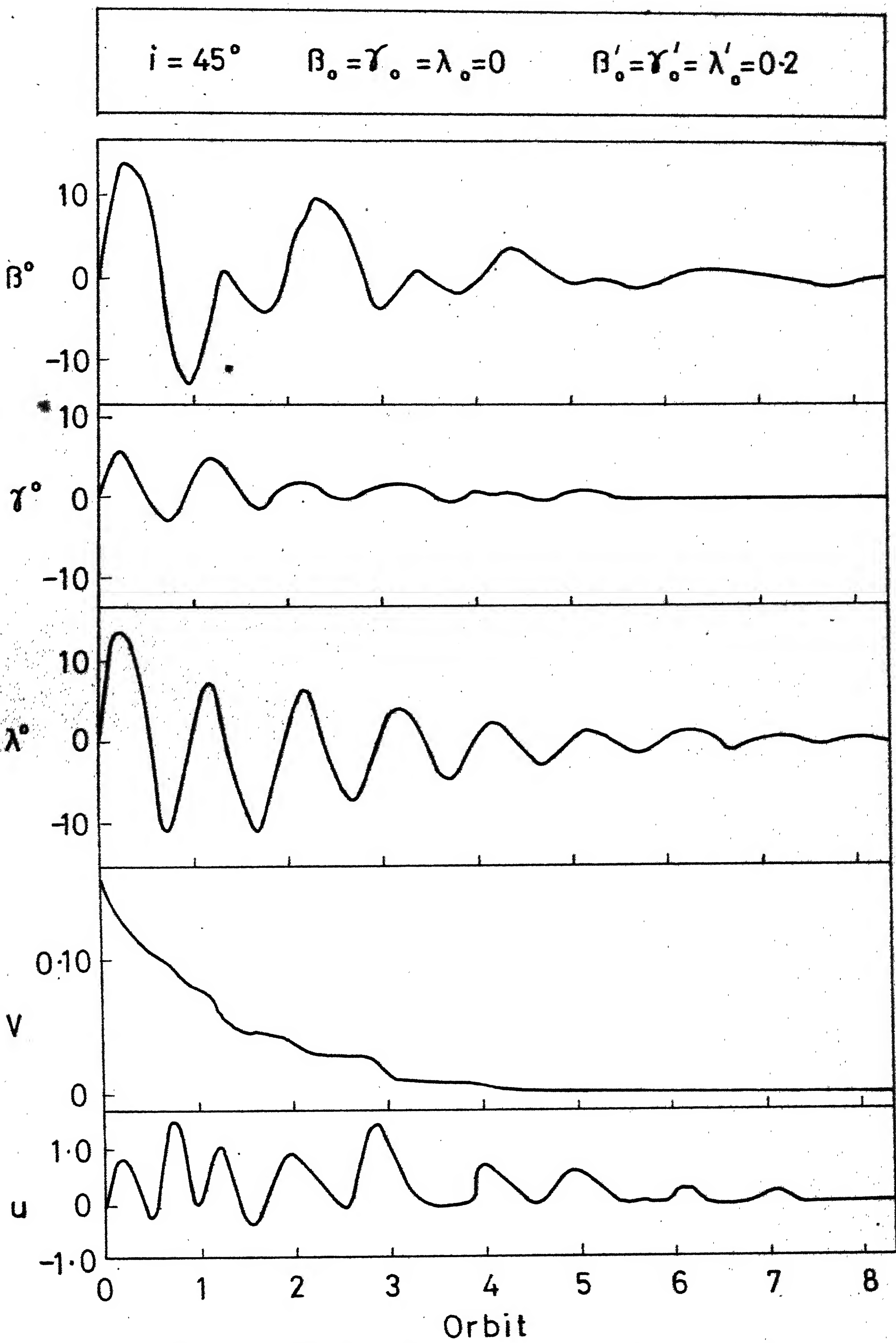
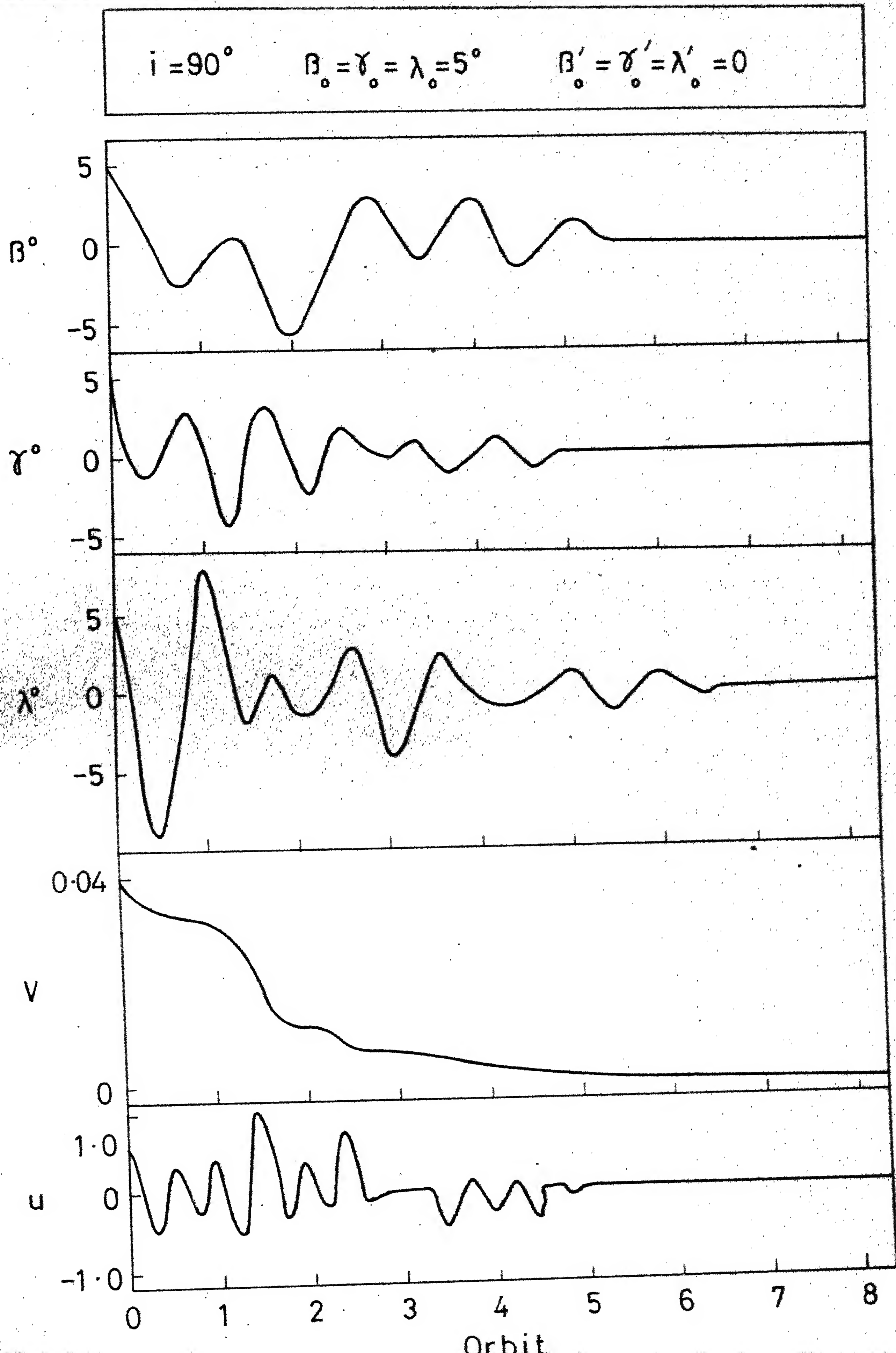
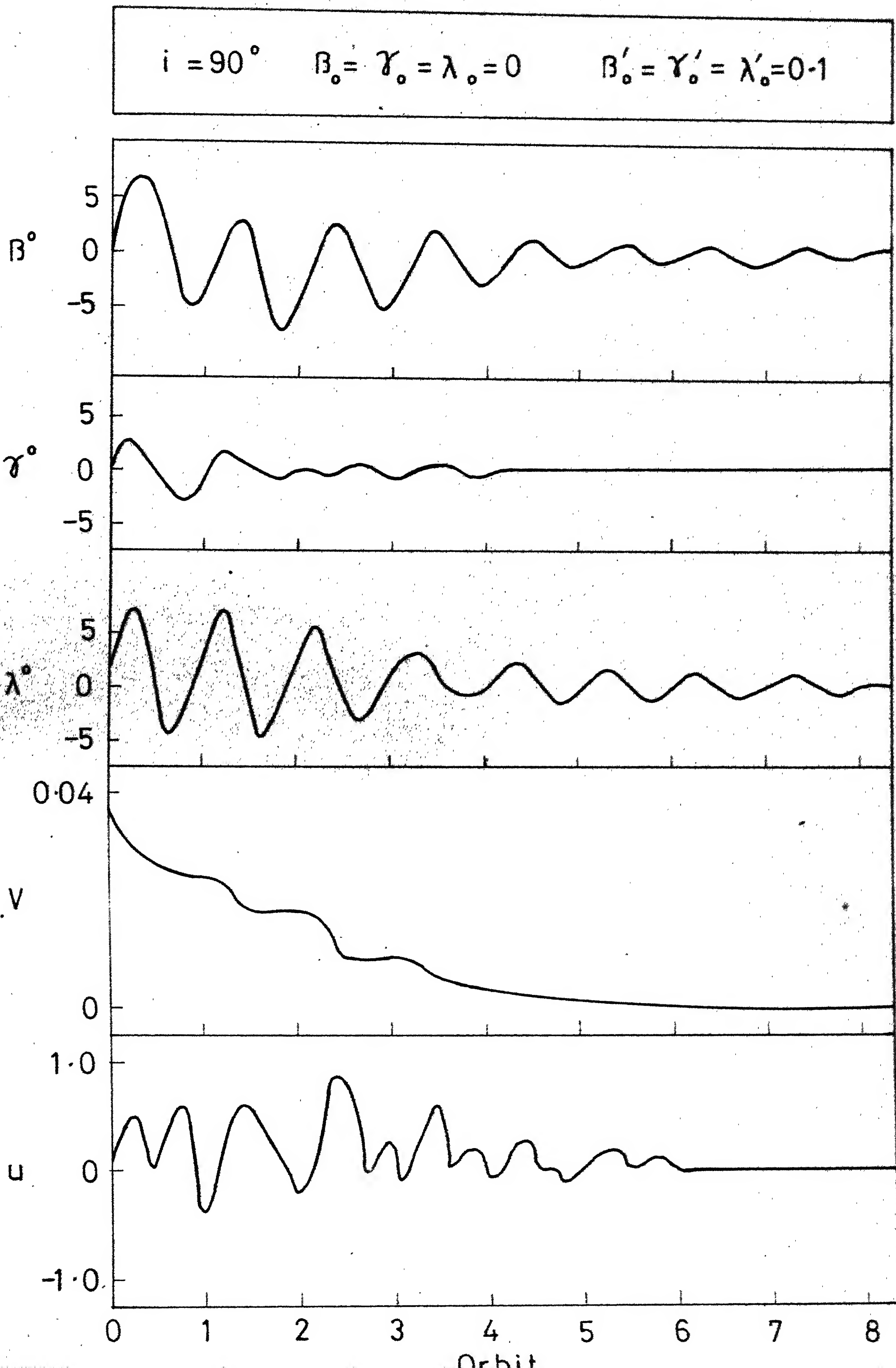


Fig.4.4 System response and control history in an inclined orbit

It was decided next to examine the effectiveness of the approach in orbits with different inclinations. The results for the case of $i = 45°$ have already been discussed. Typical responses for the case of a polar orbit ($i = 90°$) are shown in Figs. 4.5 and 4.6 for initial position and velocity disturbances, respectively. The damping rates appear to be somewhat lower than for the case of $i = 45°$. The effect of i on the behaviour, however, is not easy to isolate since the electromagnet locations δ_1, δ_2 and the inclination i together determine the nominal angle between the dipole axis and the geomagnetic induction vector $\bar{B}$ at a given orbital position. As the latter determines the distribution of the total control torque between the three degrees of freedom, the performance in any orbit would improve through an optimum choice of δ_1 and δ_2. An exception occurs for orbit very close to the equatorial plane (i=0) where the pitch component of the control torque remains small for any δ_1, δ_2 because the vector $\bar{B}$ is nominally at $\varepsilon = 11.4°$ from the pitch axis. In such a case, the overall effectiveness of the control system reduces as indicated by the responses shown in the Figs.4.7 and and 4.8. The system performance was found to be quite sensitive to the choice of K and α. It appears possible to reduce the settling times significantly through an optimum selection of these parameters. The nature of the equations being nonautonomous, an optimization study would have to



i = 90°   β₀ = γ₀ = λ₀ = 5°   β′₀ = γ′₀ = λ′₀ = 0
β°
γ°
λ°
V
0·04
u
1·0
−1·0
0 1 2 3 4 5 6 7 8
Orbit



i = 90°   ß₀ = γ₀ = λ₀ = 0   ß′₀ = γ′₀ = λ′₀ = 0·1
ß°
γ°
λ°
V
u
5
0
-5
0·04
1·0
-1·0
0 1 2 3 4 5 6 7 8
Orbit

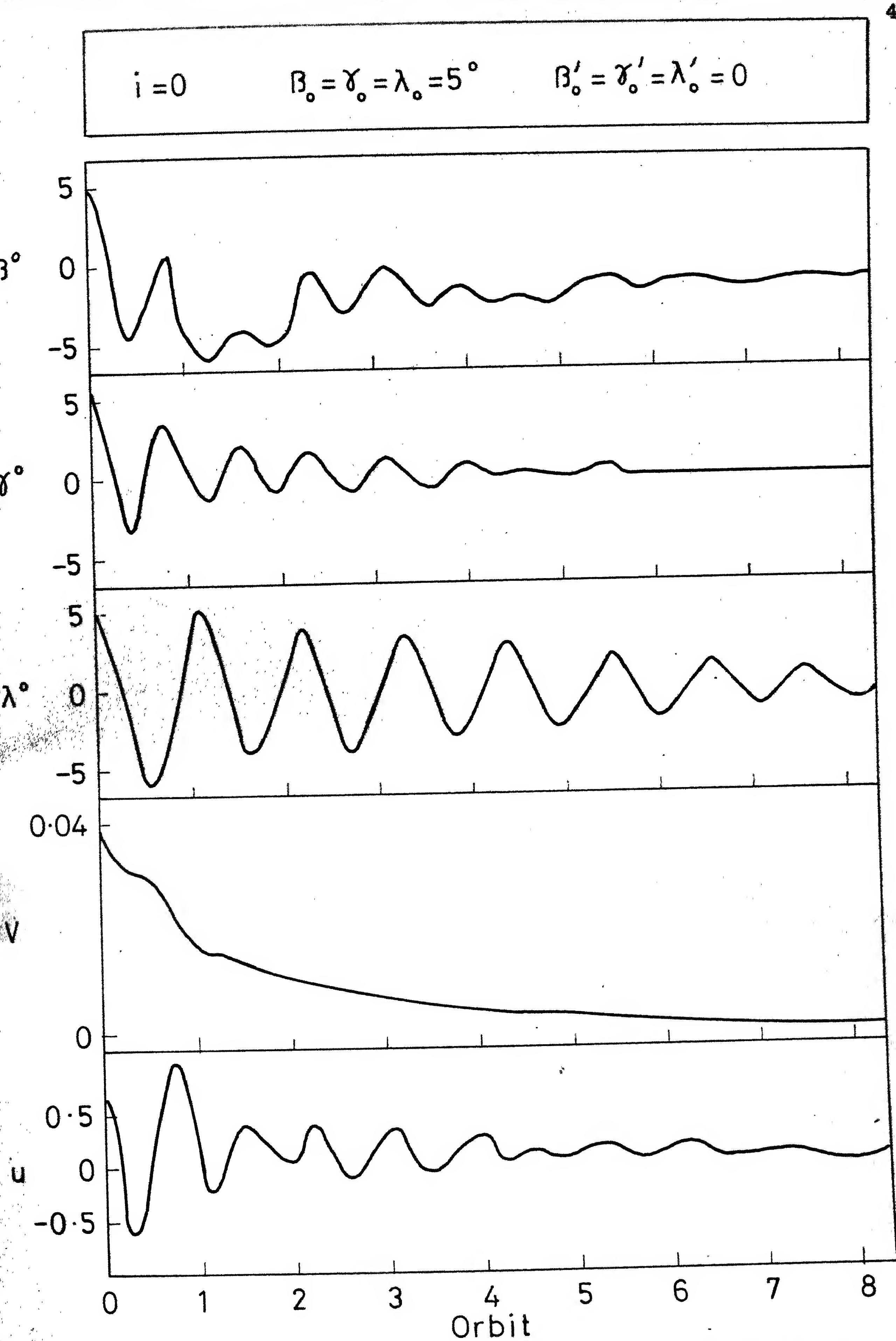


Fig.4.7 System response and control history subsequent to initial

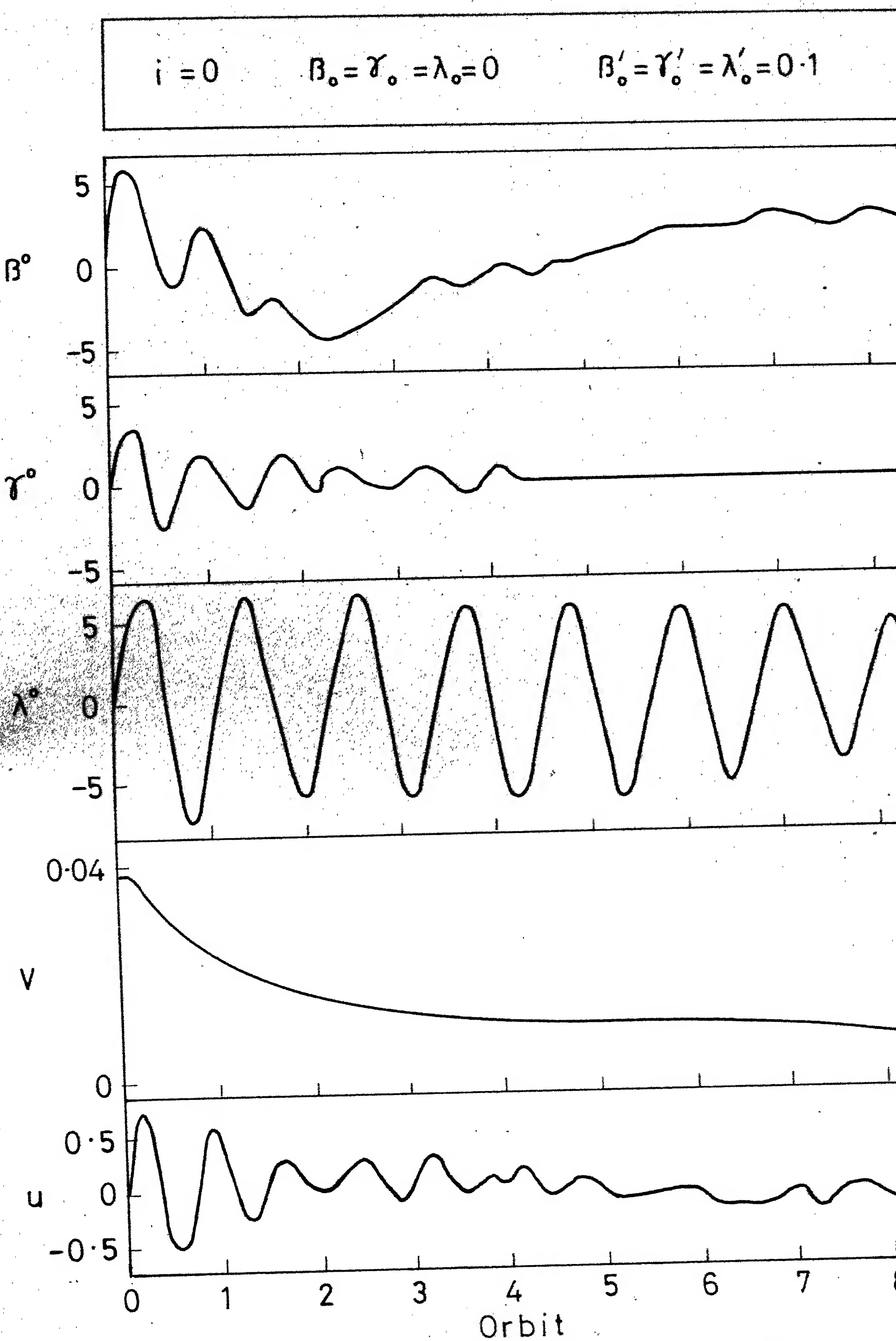


Fig.4.8 System response and control history subsequent

consider the response due to distirbances that might occur at any position in the orbit. The effort required appears worthwhile only for a specific configuration during the design stage.

Finally, a comment about the effect of the orbit altitude would be important. When the Eqns. (2.8, 2.9 and 2.10) are substituted in Eqns. (2.12), the orbit radius R is absorbed in the constant $1/\Omega^2 R^3 = 1/\mu$. The magnetic torque components then contain R explicitly. only in the terms of order ε which is small. Therefore, the non-dimensionalized responses over a range of R would remain practically the same except when $i \approx 0$.

An attempt was also made to evaluate the response of the system in conjunction with the bang-bang control law (Eqn. 3.19). With $u_o = 5$ amp-m^2, $\alpha = 1.5$ and other parameters as before, considerably faster damping was indicated by the numerical results. Unfortunately, sufficient quantitative agreement could not be found between the various computer results obtained with step sizes ranging from 1° to 0.01°. This appears to be due to the high sensitivity of the system to the switching times of the control u. Efforts to improve the quantitative agreement were found to involve a prohibitive amount of computation time.

5. CONCLUDING REMARKS AND RECOMMENDATIONS FOR FUTURE WORK

The main conclusions based on the study may be summarized as follows:

1. A magnetic controller employing a single electromagnet has been proposed which is capable of damping the roll, yaw and pitch disturbances simultaneously.

2. A feedback control law for the onboard electromagnet has been derived using Liapunov's second method of stability. The control obtained is of a general nature permitting a wide choice of the dependence of the control variable on the error function.

3. The system is found to be qite effective against both initial attitude errors as well as impulsive disturbances. Its performance may be further improved through an optimum choice of the controller parameters. The system appears suitable for applications not demanding a high speed of response, such as, long-life scientific satellites.

4. The ability of the controller in orbits with any inclination has been demonstrated, although the response is relatively slower in near-equatorial orbits.

5. The system does not involve any mass expulsion schemes. Only a small amount of electrical energy is required to activate the electromagnet which can be generated onboard

by means of solar cells. As there are no moving parts, it represents an inherently more reliable approach. Thus the controller promises an increased life-span for the spacecraft.

The following recommendations are made for future work based on the present study:

1. It would be desirable to obtain more precisely the results with the proposed bang-bang control law. This would also permit an optimization of the system performance with respect to the weighting parameter .

2. The procedure should be extended for the case of a satellite in an elliptic orbit. At high altitudes during an elliptic orbit, the earth's magnetic field weakens further while the solar radiation pressure assumes a dominant role. It appears interesting to develop a hybrid magnetic-solar controller permitting attitude control by a semi-passive approach throughout the orbit.

3. At low altitudes, both the aerodynamic forces and the earth's magnetic field are significant. A magnetic control approach could be developed to counter the continuous disturbance resulting from the aerodynamic torques.

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